Study Guide

Graph Systems of Inequalities
03/22/2012

Graph Systems of Inequalities

An inequality is a number sentence that uses *is greater than* or *is less than* symbols. For example, $6n < 4$ and $y \geq 2x - 3$ are inequalities.

When graphing an inequality, the student should mentally replace the inequality symbol with an equal sign in order to graph the inequality as an equation. Then use the table below to decide the type of line that should be used when drawing the graph.

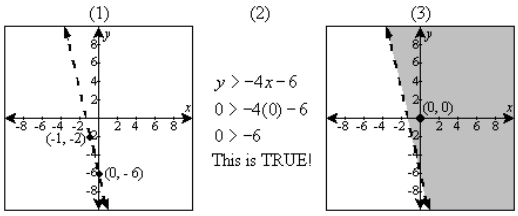
Symbol:	Definition:	Type of Line:
$>$	is greater than	Dashed
$<$	is less than	Dashed
$\geq$	is greater than or equal to	Solid
$\leq$	is less than or equal to	Solid

A dashed line tells the reader that the values on the line ARE NOT included in the inequality. A solid line tells the reader that the values on the line ARE included in the inequality.

Example 1:

Graph the inequality.

$y > -4x - 6$



Step 1: Graph the line that is represented by the inequality. (Remember to mentally replace the > with =.) This equation is given in $y = mx + b$ form (slope-intercept form), where m is the slope and b is the y-intercept. Plot the y-intercept, (0, - 6), then use the slope, - 4, to move up 4 units and to the left 1 unit. The *is greater than* symbol (>) is used, refer to the chart above to see that this symbol requires a dashed line. Connect the two points using a dashed line.

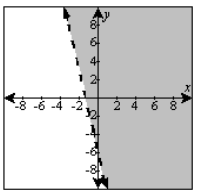
Step 2: Choose a test point (that is not on the line) to determine which side of the line should be shaded. The most common test point to use is (0, 0), but it does not matter what point is used. Substitute the test point into the inequality and simplify.

- **If the test point makes the inequality true, shade the side of the line that includes the test point.**
- **If the test point makes the inequality false, shade the side of the line that does not include the test point.**

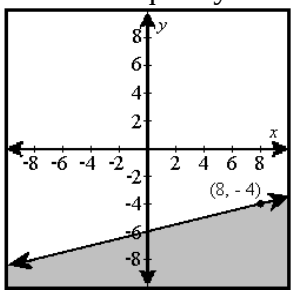
In this case, the test point makes the inequality true.

Step 3: Since the test point makes the inequality true, shade the side of the dotted line that includes the point (0, 0).

Answer:



Example 2:
Determine the correct inequality for the graph below.



(1) $y = \frac{1}{4}x - 6$

(2) $y \le \frac{1}{4}x - 6$ OR $y \ge \frac{1}{4}x - 6$

(3)

$y \le \frac{1}{4}x - 6$	OR	$y \ge \frac{1}{4}x - 6$
$-8 \le \frac{1}{4}(0) - 6$		$-8 \ge \frac{1}{4}(0) - 6$
$-8 \le -6$		$-8 \ge -6$
TRUE		FALSE

Step 1: Determine the equation of the line. In this case, the *y*-intercept is at (0, - 6) and the slope appears to be *up 1, over 4* (or ?), as can be seen by points at (8, - 4) and (4, - 5). Therefore, the equation of the boundary line is $y = (?)x - 6$.

Step 2: Use the table on page 1 to determine which type of inequality symbol to use (<, >, $\le$, or $\ge$). The line on the graph is solid, so the $\le$ or $\ge$ symbol must be used.

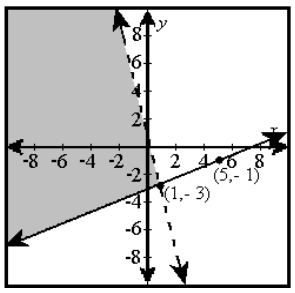
Step 3: Choose a test point from the shaded side of the line, and substitute it into each inequality to determine which of the two inequalities is correct. A good test point to use is (0, - 8), since (0, - 8) is included in the shaded area of the graph. Since $y \le (?)x - 6$ is true when (0, - 8) is used as the test point, it is the correct inequality.

Answer: $y \le \frac{1}{4}x - 6$

Systems of Inequalities:

When graphing a system of inequalities, the process is very similar. A system of inequalities is two or more inequalities. The main difference is that the final solution is **the area where the shaded regions overlap**.

Example 3:
Choose the system of inequalities represented by the following graph.



(1)
$y = \frac{2}{5}x - 3$
$y = -4x + 1$

(2)
$y \le \frac{2}{5}x - 3$ OR $y \ge \frac{2}{5}x - 3$ $y < -4x + 1$ OR $y > -4x + 1$

(3)
$y \le \frac{2}{5}x - 3$ OR $y \ge \frac{2}{5}x - 3$ $y < -4x + 1$ OR $y > -4x + 1$

$0 \le \frac{2}{5}(-2) - 3$ $0 \ge \frac{2}{5}(-2) - 3$ $0 < -4(-2) + 1$ $0 > -4(-2) + 1$

$0 \le -3\frac{4}{5}$ $0 \ge -3\frac{4}{5}$ $0 < 9$ $0 > 9$

FALSE TRUE TRUE FALSE

Step 1: Determine the equations of the boundary lines. The solid line has a *y*-intercept of - 3 and a slope of 2/5. The dashed line has a *y*-intercept of 1 and a slope of - 4. The equations of the boundary lines are $y = (2/5)x - 3$ (solid line) and $y = -4x + 1$ (dashed line).
Step 2: Use the chart on page 1 to determine which inequality symbols to use.
Step 3: Choose a test point from the shaded region of the graph and substitute it into each of the inequalities. A good test point to use is (-2, 0). Since (-2, 0) is in the shaded area of the graph, the inequalities that are true when (-2, 0) is substituted are the correct inequalities.

Answer: $y \ge \frac{2}{5}x - 3$
$y < -4x + 1$

Systems of Inequalities That Are Not Solved For *y*:
Sometimes systems of equations or inequalities are presented in a form other than y = *mx* + *b*. If an inequality is not in this form, the student should first solve the inequality for *y* in order to make the graphing process easier.
Remember, to solve an inequality for *y*, use inverse operations to isolate the variable and be sure to follow inequality sign rules when multiplying or dividing.

Rules:
•When multiplying or dividing both sides of an inequality by a positive number, leave the inequality sign as is.
•When multiplying or dividing both sides of an inequality by a negative number, reverse the direction of the inequality sign.
•When adding or subtracting both sides of an inequality by a positive or negative number, leave the inequality sign as is.

Example 4:
Rewrite the following system of inequalities in order to solve for *y*. For this example you do not need to actually graph the system.

$8x - 4y > 24$
$5 \le 6y + x$

(1)
$-4y > -8x + 24$ $5 - x \le 6y$

(2)
$\frac{-4}{-4}y > \frac{-8x}{-4} + \frac{24}{-4}$ $\frac{5}{6} - \frac{1}{6}x \le \frac{6y}{6}$

(3)
$y < 2x - 6$ $\frac{5}{6} - \frac{1}{6}x \le y$

Step 1: Isolate the *y*-term by adding or subtracting the *x*-term.
Step 2: In order to isolate the variable in the first inequality, it is necessary to divide both sides of the

inequality by a - 4. In the second inequality, both sides need to be divided by 6.
Step 3: Simplify each term in the inequality. Remember, since the first inequality was divided by a negative number, the direction of the inequality sign must be changed. In the second inequality, the sign remains the same since the division was by a positive number.

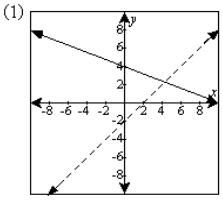
$y < 2x - 6$

nswer: $\frac{5}{6} - \frac{1}{6}x \le y$

Example 5:
Graph the solution to the following system of inequalities.

$y \le -\frac{1}{3}x + 4$

$y > x - 2$

(1) 

(2) $y \le -\frac{1}{3}x + 4$

$0 \le -\frac{1}{3}(0) + 4$

$0 \le 4$

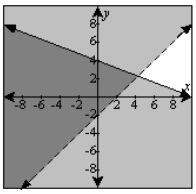
TRUE

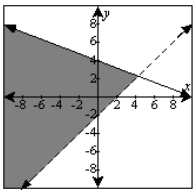
(3) $y > x - 2$

$0 > 0 - 2$

$0 > -2$

TRUE



(4) 

Step 1: Graph the lines that are represented by the inequalities. The *y*-intercept of the top line is 4 and the slope is -1/3. The *y*-intercept of the second line is -2 and the slope is 1. Use the chart on page 1 to determine whether the lines should be dashed or solid. The line with the $\le$ symbol should be solid, and the line with the > symbol should be dashed.
Step 2: Choose a test point, and substitute it into both inequalities to determine which direction to shade. It does not matter which point is used as a test point as long as the correct side of the line is shaded. A good point to use is (0, 0).
Step 3: Since the point (0, 0) makes both inequalities true, shade each inequality on the side of the line that contains (0, 0).
Step 4: The solution to the two inequalities is the region of the graph where the shading overlaps.

Answer: